Temperature Dependence of Indirect Interband Tunneling in Germanium*

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The temperature dependence of the tunneling current in antimony-doped germanium tunnel diodes has been measured between 1.2 and 360°K at a number of bias voltages. The results have been compared with theory of Kane in a manner which is insensitive to the lack of agreement between the observed I–V characteristic and the theoretically predicted shape. All important effects have been taken into account including the explicit temperature dependences of the band gap, the effective masses, the phonon densities, the Fermi functions, and of the Fermi level positions with respect to the band edges. Excellent agreement was obtained both as to the temperature dependence itself and also to its bias dependence. The magnitude of the tunneling exponent could be determined from the measurements. This value agrees with that obtained from the effect of stress on the same tunnel diodes. Using the value 4×10^{-49} erg^2 cm^3 for the electron-phonon coupling constant which has been estimated from other experiments, the absolute magnitude of Kane's expression is found to be too small by a factor of about 20. Some difficulty is noted with respect to the phonon energy terms in the tunneling exponents of the theoretical expression.

I. INTRODUCTION

INTERBAND tunneling has been a fruitful area of study not only for its own sake, but also as a tool for studying the properties of semiconductors. Some parameters which can be measured only with great difficulty by other means can be determined quite simply from the properties of the appropriate tunnel diode. For example, by measuring the structure of the I-V characteristic of tunnnel diodes at 4.2°K, Holonyak *et al.*[1] deduced the energies of the phonons whose wave number corresponds to the indirect band gap in germanium and silicon. The same technique was employed by Hall *et al.*[2] to deduce the electron-phonon coupling constant in polar semiconductors. These investigations, as well as others[3,4] have been based on the interpretation of rather distinctive features of the I-V characteristic at helium temperatures. As can be seen in Fig. 1, these features unfortunately disappear very rapidly with increasing temperature. This fact not only makes it very difficult to follow the temperature dependence of the phenomena which give rise to the structure at low temperatures, but also implies that a different approach is needed to study any other phenomenon which may become important at higher temperatures. Before any headway can be made, it is clearly necessary to have a rather detailed understanding of the temperature dependence. In this paper, we shall attempt a step in this direction by providing a detailed quantitative comparison of the theoretical expression derived by Kane based on the theory of indirect tunneling given by Keldysh[5] with the experimental results for antimony-doped germanium tunnel diodes.

The important effects which should give rise to the temperature dependence have been known for some time. Esaki[6] pointed out and presented experimental evidence that the decrease of the tunnel current with increasing temperature in phosphorus-doped germanium tunnel diodes can be understood on the basis of the broadening out of the Fermi function. In addition to this effect, which is most important in lightly doped diodes and at larger forward biases, one expects the tunneling probability (assumed constant by Esaki) to vary through the variation of the band gap with temperature. In the case of phonon assisted tunneling one also expects that the tunneling current will depend on the number of thermal phonons present.

Chynoweth *et al.*[7] in connection with their experiments on Ge and Si tunneling diodes, discussed and demonstrated the importance of the temperature dependence of the energy gap and of the density of phonons needed for indirect tunneling. Their analysis, however, demonstrates only the gross features of the temperature dependence of the tunneling current without constituting a fair comparison with the theory.

The temperature dependence of As-doped Ge diodes was investigated by Meyerhofer *et al.*[8] In the case of very heavily doped tunnel diodes, they observed the

* A part of the research reported in this paper was sponsored by the U. S. Air Force Office of Scientific Research through grants AFOSR 62-178 and AFOSR 148-63.

[1] H. Holonyak, Jr., I. A. Lesk, R. N. Hall, J. J. Tiemann, and H. Ehrenreich, Phys. Rev. Letters 3, 167 (1959).

[2] R. N. Hall, J. H. Racette, and H. Ehrenreich, Phys. Rev. Letters 4, 456 (1960).

[3] J. V. Morgan and E. O. Kane, Phys. Rev. Letters 3, 466 (1959).

[4] A. G. Chynoweth, G. H. Wannier, R. A. Logan, and D. E. Thomas, Phys. Rev. Letters 5, 57 (1960).

[5] E. O. Kane, J. Appl. Phys. 32, 83 (1961); L. V. Keldysh, Zh. Eksperim. i Teor. Fiz. 34, 962 (1958) [translation: Soviet Phys.—JETP 34, 665 (1958)].

[6] L. Esaki, Phys. Rev. 109, 603 (1958).

[7] A. G. Chynoweth, W. L. Feldman, C. A. Lee, R. A. Logan, G. L. Pearson, and P. Aigrain, Phys. Rev. 118, 425 (1960).

[8] D. Meyerhofer, G. A. Brown, and H. S. Sommers, Phys. Rev. 126, 1329 (1962).

expected exponential dependence of the current on the band gap variation. Using the temperature variation of the direct gap for pure germanium, they found a value for the exponent of the tunneling probability which is in fair agreement with Kane's expression for direct tunneling when a reasonable choice of the impurity distribution in the junction is made. They did not calculate the effect of the broadening of the Fermi functions, however, and a detailed comparison with the theory was not attempted. Furthermore, serious objections can be raised against treating the phonon-unassisted tunneling in As-doped germanium with the simple theory of direct tunneling. The direct-type tunneling in As-doped germanium, which is an indirect gap semiconductor, requires a special treatment which takes into account the interaction of the electrons with the strong central cell potentials of the impurities. Until the mechanism for this effect has been established, and its temperature dependence has been calculated, these results cannot be considered a decisive test of the theory. In the present work, Sb-doped germanium was chosen as the base material for the diodes because indirect tunneling is observed in this material in its purest form.

It will be seen that the temperature dependence of the tunneling current depends strongly on the numerical value of the exponent in the expression for the tunneling probability. Since this quantity depends critically on the details of the impurity profile across the junction region, it is very difficult to calculate with good accuracy. We therefore measured the temperature dependence of the same diodes which were previously used for the study of the effect of strain on interband tunneling.[9] In Ref. 9, it was shown that the effect of elastic strain depends on the exponent in a very similar manner. The present work therefore serves as an independent check on Ref. 9, and vice versa.[10]

The manner in which the variations of the electron distribution functions and the band gap should be taken into account have been adequately discussed in the literature.[6–8,11] The proper treatment of the phonon amplitudes, however, has not been discussed for $T>0$. In incorporating the phonon-Bose factors into the indirect tunneling-current expression one has to consider that the measured current at a particular bias is the difference between two electron currents flowing across the junction in opposite directions. Each of these currents consists of two contributions, one involving phonon emission, the other phonon absorption. For each kind of phonon the integral over the electron energy states for these four current contributions have to be evaluated and multiplied with their respective Bose factors, electron-phonon coupling constants, and tunnel probabilities before they are summed to yield the total current. The effects of both the TA and the LA phonons are included in the present analysis exactly. The optical phonons are also included in an approximate manner.

We have measured the tunnel current at a number of bias values that are held fixed while the temperature was varied between 1.2 and 360°K. The results over the entire range of temperature are compared with the indirect tunneling theory of Kane[5] taking into account the temperature dependence of (a) the energy gap and effective masses, (b) the Fermi functions, (c) the position of the Fermi levels with respect to the band edges, and (d) the phonon densities in the manner outlined above.

Our comparison is insensitive to the way the junction field is assumed to vary with bias. It appears to be quite insensitive also to the details of the shape of the I-V characteristic. This is advantageous in that our conclusions are not affected by the lack of agreement between the observed I-V characteristic and the theoretically predicted shapes, but it is unfortunate in that we are not able to distinguish between any of the theoretical expressions presented so far.[12]

II. EXPERIMENTAL PROCEDURE AND RESULTS

The cryostat was of conventional design. To assure temperature equilibrium the sample was placed inside a copper container which in turn was thermally connected with 150 g of granular charcoal. This acted, by virtue of its large heat capacity at low temperatures, as a thermal reservoir, and it stabilized the temperature of the sample at values above 4.2°K. The temperature was measured with a carbon resistor mounted in good thermal contact with the sample.

The bias was measured potentiometrically using double contact wires connected both to the diode dot and to the Ohmic contact. The current was measured as a function of temperature for fixed bias values. It was found necessary to eliminate ac pickup which can cause erroneous voltage readings. These errors are especially troublesome near the phonon thresholds where the diode characteristic is particularly nonlinear.

The current-voltage characteristic of a Sb-doped Ge diode is shown in Fig. 1 for the temperatures 4.2 and 78°K. At 4.2°K one observes the structure in the I-V characteristic which is typical for indirect tunneling. This material displays an extremely low conductance at biases below that needed to emit the lowest energy

[9] H. Fritzsche and J. J. Tiemann, Phys. Rev. **130**, 617 (1960).

[10] In Ref. 9 an error was made in evaluating the theoretical exponent for direct tunneling. The first two values of α in Table I should be multiplied by $\sqrt{2}$. Thus, for the theoretical value of α, 8.5 should be replaced by 12.0, while the value discussed in the text becomes 17.0. None of the other values are changed. The agreement between theory and experiment is therefore much better than was indicated. The discussion of the bias dependence of β, which occurs prior to Eq. (14) is also affected by the error. At $V=-300$ mV, the theoretically derived value of β is 18.8 rather than 26.5.

[11] T. A. Longo, Bull. Am. Phys. Soc. **5**, 160 (1960); A. Blicker, R. M. Minton, and R. Glicksman, Proc. IRE **49**, 1428 (1961); Y. Furukawa, J. Phys. Soc. Japan **15**, 1130 (1960).

[12] An expression for indirect tunneling wherein the bands are coupled by the electric field at $\mathbf{k}=0$ (direct tunneling), and the conduction band states at the band edge are coupled to those at the zone center by the electron-phonon interaction has been derived by Kan Tzuchao, Acta Phys. Sinica **19**, 49 (1963).

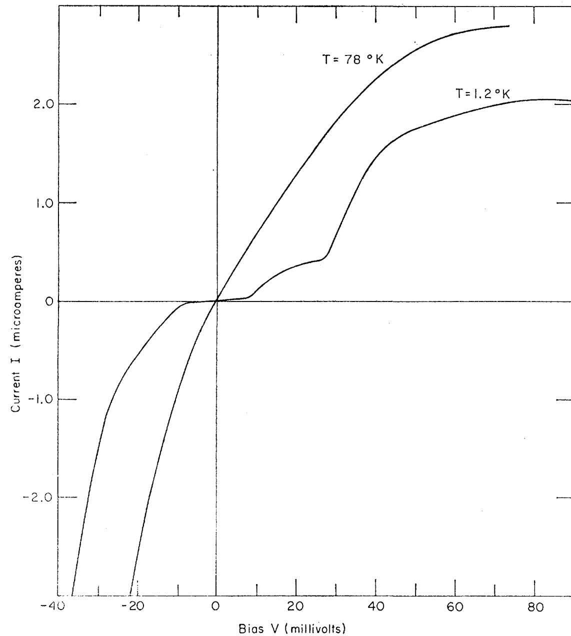

FIG. 1. Current-voltage characteristic of an antimony-doped germanium tunnel diode at 1.2 and 78°K. Note the structure observed at 1.2°K which is caused by the onsets of the TA and LA phonon contributions near $V=\pm 7.6$ and ± 28 mV, respectively. This structure is absent at 78°K.

phonon of the appropriate wave number. This fact demonstrates that the excess current, any Ohmic leakage across the junction, and the direct current (which dominates in As-doped germanium diodes) are negligibly small compared to the indirect-tunneling current whose temperature dependence we wish to study. It is seen from Fig. 1 that phonon absorption is already strong enough at 78°K to wash out the phonon structure in the I-V characteristic.

In order to compare the temperature dependence of the indirect-tunnel current with the predictions of the theory we plotted the fractional change of the current, $\Delta I/I_0=[I(T)-I(0)]/I(0)$, for various biases as a function of temperature in Figs. 2, 3, and 4. This quantity has the advantage that the comparison is independent of the junction area and of absolute magnitude factors which are uncertain in the theoretical expression. Furthermore, this method of comparison is less sensitive to the detailed shape of the I-V characteristic. This insensitivity is desirable because the I-V characteristic is not accurately reproduced by the theory in its present form. This point will be discussed later.

In plotting $\Delta I/I_0$ the current at $T=1.2$°K was used as $I(0)$. No significant change occurred between 1.2 and 4.2°K. The bias voltages chosen for the theoretical comparison are larger than the onset voltage for the 0.0076 eV phonon in both forward and reverse directions.

The data presented in Figs. 1–4 were obtained from sample 1 of Ref. 9. The other diodes and samples used in that work show a fractional change of the tunneling current with temperature in agreement with that reported here within the experimental error of 5%.

From Hall coefficient measurements the Fermi level penetration on the n-type side of these diodes was determined to be $\zeta_n=0.020\pm 0.002$ eV. The Fermi level on the p-type side was estimated to be 0.140 ± 0.020 eV.

III. COMPARISON WITH THEORY

According to Kane[5] the current density for indirect tunneling between one conduction band valley and the valence band is

$$j=\sum_i C\{(N_i+1)[P_i(fe)S_i(fe)-P_i(re)S_i(re)] + N_i[P_i(fa)S_i(fa)-P_i(ra)S_i(ra)]\}. \quad (1)$$

The sum extends over the i different phonons of energy $\hbar\omega_i$ which have the wave number corresponding to the difference in $\mathbf{k}$ between the (000) valence band and the

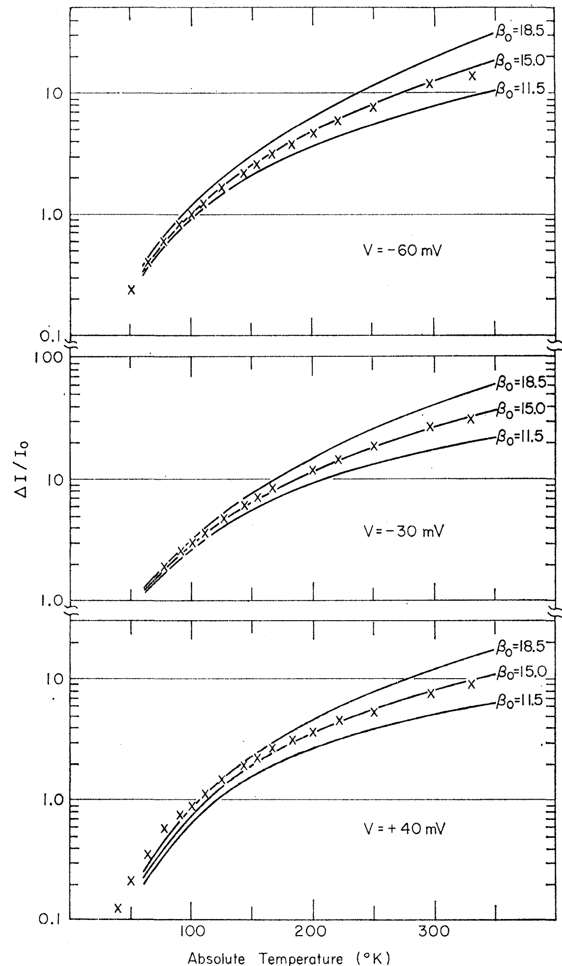

FIG. 2. The relative current change $[I(T)-I(0)]/I(0)$ as a function of absolute temperature for various bias values. The crosses represent experimental points. The solid curves were calculated on the basis of Eq. (1) [see text]. The parameter $\beta_0=15.0$ yields the best fit to the data.

(111) conduction-band valleys. The quantity C is given by Kane as

$$C=\frac{e(\bar{E}_{\perp})^{2}(m_{de})^{3/2}(m_{dh})^{3/2}}{4(2^{3/4})\pi^{5/2}\hbar^{13/2}m_{rx}{}^{1/4}E_{g}{}^{3/4}F^{1/2}},\tag{2}$$

where m_{de} and m_{dh} are the density-of-states masses for electrons and holes, respectively, m_{rx} is the reduced mass in the tunneling direction, F is the average junction field chosen to give the same penetration factor as the actual field, E_g is the indirect band gap, and N_i the phonon occupation number for the ith phonon. The quantity $\bar{E}_{\perp}$ is defined as

$$\bar{E}_{\perp}=\hbar F/2^{3/2}m_{rx}{}^{1/2}E_{g}{}^{1/2}.\tag{3}$$

In Eq. (1) the symbol f, (r), stands for forward, (reverse), or for electron tunneling from n to p, (p to n); the symbols e and a stand for phonon emission and absorption, respectively. The tunneling probabilities[13] are given by

$$P_i(fe)=P_i(ra)=B_i(fe)\times\exp[-\tfrac{2}{3}(E_g/\bar{E}_{\perp})(1-\hbar\omega_i/E_g)^{3/2}]$$
$$P_i(re)=P_i(fa)=B_i(re)\times\exp[-\tfrac{2}{3}(E_g/\bar{E}_{\perp})(1+\hbar\omega_i/E_g)^{3/2}].\tag{4}$$

The B_i's are the electron-phonon coupling constants for the ith phonon. Because of the condition of detailed balance they also obey the relations

$$B_i(fe)=B_i(ra),$$
$$B_i(re)=B_i(fa).$$

The quantities $S_i(fe)$, etc., in Eq. (1) are integrals over the electron distribution functions and the tunneling attenuation factor that results from the conservation of perpendicular momentum. For example, in Kane's notation

$$S_i(fe)=\int f_1(E_1)[1-f_2(E_2)][1-\exp(-E_1/\bar{E}_{\perp})]\times[1-\exp(-E_2/\bar{E}_{\perp})]dE$$
$$S_i(re)=\int f_2(E_2)[1-f_1(E_1)][1-\exp(-E_1/\bar{E}_{\perp})]\times[1-\exp(-E_2/\bar{E}_{\perp})]dE.\tag{5}$$

These integrals differ from Kane's D integral in that the electron flow in each direction is treated separately. The energies E_1 and E_2 are measured from the band edges on the n-type and p-type side, respectively. They refer to an initial and final state which are separated in energy by $\pm\hbar\omega_i$ depending on whether phonon absorption or emission is being considered. It should be noted that at zero temperature the S-functions intersect the $I=0$ axis at $V=\pm\hbar\omega_i/e$ rather than at $V=0$.

[13] Some other prefactors which belong in the tunneling probability but which are independent of the phonons have been absorbed in the quantity C for simplicity.

At zero temperature, Eq. (1) reduces to the expression given by Kane because the Fermi functions f_1 and f_2 are then step functions and the current flows in one direction only at each bias voltage. For finite temperatures, however, Kane's D function must be split into the two parts which correspond to current flows in opposite direction because the tunneling probabilities for these currents are different [see Eq. (4)]. The temperature dependence of the Fermi level penetrations on the n-type and p-type sides is obtained from the condition that the total electron and hole concentrations are independent of temperature.

A. Assumptions

The important assumptions in our calculation are the following:

(1) Although the comparison of $\Delta I/I_0$ with theory does not require knowledge of the absolute magnitudes of the electron-phonon coupling constants, the results are in fact sensitive to their sizes relative to each other. The three independent ratios $P_1(fe):P_1(re):P_2(fe):P_2(re)$ were determined from the changes in the slope of the I-V characteristic near the phonon thresholds measured at 1.2°K. These ratios were then assumed to be independent of temperature. The ratios were found to be 1:1.0:3.07:3.07.

(2) For the temperature variation of the indirect energy gap the results of McFarlane *et al.*[14] for pure germanium were taken.

(3) In this calculation we considered only the 0.0076-eV TA phonon and the 0.028-eV LA phonon.[14] The coupling constants for the optical phonons are relatively weak and they contribute very little to the total current.[1] Furthermore their energies are only slightly larger than those of the LA phonon.[15] They were taken into account approximately by adding their coupling constants to that of the LA phonons. Hence, there is a slight error at the higher temperatures because the optical modes were assumed to have the same energy as the LA phonon.

(4) The quantity C as defined by Eq. (2) was assumed to be temperature-independent. This assumption is not important at the lower temperatures, and it underestimates $\Delta I/I_0$ by 12% at 300°K.

(5) The contributions from the different conduction band valleys were assumed to be equal. Hence, Eq. (1) was multiplied by 4 to yield the total current density. The data presented here are for a diode with the junction field along [100]. Therefore, this procedure is correct in this case. In general, the conduction band valleys will have different effective mass projections along the field directions. Experimentally we observed, however, no orientation dependence of $\Delta I/I_0$. This

[14] G. G. MacFarlan, T. P. McLean, J. E. Quarrington, and V. Roberts, Phys. Rev. **108**, 1377 (1957).

[15] B. N. Brockhouse, J. Phys. Chem. Solids **8**, 400 (1959).

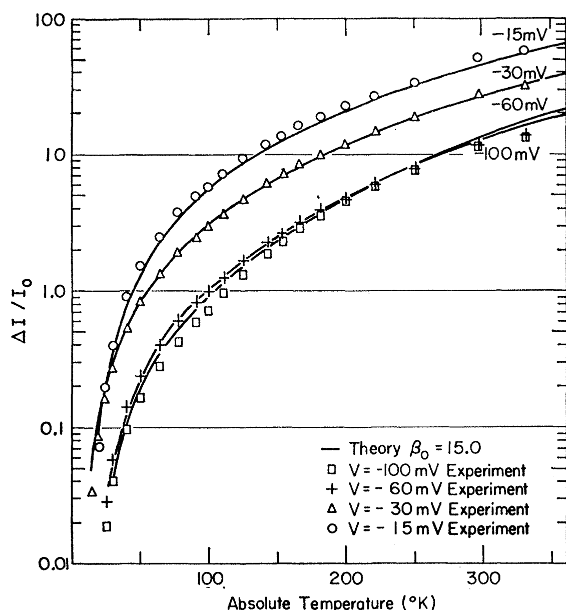

FIG. 3. The relative current change as a function of absolute temperature for several negative bias values. The solid lines were calculated with the parameter $\beta_0=15.0$. Note how well the theory reproduces the bias dependence of $\Delta I/I_0$.

indicates that the effective mass anisotropy has a negligible effect on the temperature dependence.

(6) The energy bands were assumed to be parabolic. There is considerable evidence that the masses of both the electrons and the holes increase as their energy increases. Perhaps Kane's theory can be modified by calculating the tunneling probability with the energy-dependent effective masses and including this factor before integrating over the electron distribution function. This would give rise to an additional bias dependence, particularly in the reverse direction.

(7) The explicit temperature dependence of the effective masses has not yet been measured experimentally. This dependence was, therefore, obtained from theoretical arguments outlined in the following subsection.

B. Variation of Effective Masses with Temperature

Vasileff[16] has shown that the variation with temperature of the effective masses caused by lattice vibrations via the harmonic part of the interatomic forces is negligibly small. To a very good approximation, therefore, the entire effect is due to the volume change which accompanies the change in temperature.

This effect can be estimated from hydrostatic measurements through the relation[17]

$$\frac{1}{m}\frac{\partial m^*}{\partial T^*}\bigg|_P = -\frac{3\alpha}{K}\frac{1}{E_g^*}\frac{\partial E_g^*}{\partial P}\bigg|_T, \tag{6}$$

[16] H. D. Vasileff, Phys. Rev. **105**, 441 (1957).
[17] H. Ehrenreich, J. Phys. Chem. Solids **2**, 131 (1957).

where α is the coefficient of linear expansion, K is the bulk compressibility, and E_g^* is the effective band gap for the effective mass. For the transverse electron mass, E_g^* is the gap at the (111) zone face at absolute zero temperature, and for the light hole mass, E_g^* is the band gap at the zone center. Thus,

$$\frac{\Delta m_h^*}{m_h^*\Delta T} = -\frac{3\alpha}{K}\frac{\Delta E_g(000)}{E_g(000)\Delta P},$$

and

$$\frac{\Delta m_e^*}{m_e^*\Delta T} = -\frac{3\alpha}{K}\frac{\Delta E_g(111)}{E_g(111)\Delta P}. \tag{7}$$

The pressure coefficients at the zone center and at the (111) edge of the Brillouin zone have been measured by Paul[18] and by Zallen *et al.*,[19] respectively, and the quantities α and K have been measured by Fine.[20]

Using the values

$$\frac{\Delta E_g(000)}{E_g(000)\Delta P} = \Xi_{(000)} = 12.4\times10^{-12}\ \mathrm{cm^2/dyn},$$

$$\frac{\Delta E_g(111)}{E_g(111)\Delta P} = \Xi_{(111)} = 3.4\times10^{-12}\ \mathrm{cm^2/dyn},$$

and Fine's results, we find that the temperature variation of the reduced effective mass is very nearly proportional to the change in the band gap itself and that the relative change is four times smaller. That is

$$\Delta m_r/m_r = \tfrac{1}{4}(\Delta E_g/E_g).$$

Since the effective mass itself enters into Kane's expression in a less sensitive manner than the band gap, the above treatment of the temperature variation of the reduced effective mass is quite satisfactory.

C. Comparison with Experimental Data

For the comparison with the experimental results the relative change of the total tunneling current $\Delta I/I_0$ was evaluated from Eq. (1) by means of a computer. The calculation was carried out for various values of the factor

$$\beta_0 = \tfrac{2}{3}(E_g/\bar{E}_\perp)_{T=0}, \tag{8}$$

which is approximately equal to the exponent of the tunneling probability in Eq. (4). The results of the calculation are represented by the solid curves in Fig. 2 for three bias values. At higher temperatures $\Delta I/I_0$ depends quite strongly on the choice of β_0. This enables us to determine β_0 from the best fit between the theory and the experimental data. We obtain $\beta_0=15.0\pm1$ which is in good agreement with the stress measurements on the

[18] W. Paul, J. Phys. Chem. Solids **8**, 196 (1959).
[19] R. Zallen, W. Paul, and J. Tauc, Bull. Am. Phys. Soc. **7**, 185 (1962).
[20] M. E. Fine, J. Appl. Phys. **24**, 338 (1953); **26**, 862 (1955).

same tunnel diodes.[9] Figures 3 and 4 compare the theoretical curves based on the value $\beta_0=15.0$ with the experimental results for several reverse and forward bias values. Except for the large forward bias of +70 mV the theory agrees with the experiments to within 20%. The shape of the temperature dependence of $\Delta I/I_0$ and the strong increase of $\Delta I/I_0$ with decreasing magnitude of the bias is reproduced very well by the theory.

Before we interpret the observed discrepancies between the theory and the experiment, it will be helpful to estimate the relative importance of the various factors which contribute to the temperature dependence of the tunneling current. Since the machine calculations cannot be easily analyzed, we shall present and discuss in the following an approximate calculation which becomes valid at large negative biases.

First, let us ignore the phonon energies in the tunneling probabilities and take

$$P_i/B_i=\exp[-\tfrac{2}{3}(E_g/\bar{E}_\perp)]. \tag{9}$$

If we further restrict our attention to large negative bias values where the phonon structure is not important, the current density then simplifies to

$$j=CD(V,T)\exp\left[-\frac{2}{3}\frac{E_g(T)}{\bar{E}_\perp(T)}\right]\sum_i B_i(2N_i+1), \tag{10}$$

where $D(V,T)$ is Kane's D function calculated without considering the phonon energies. The B_i is a measure of the coupling strength of the ith phonon. The relative

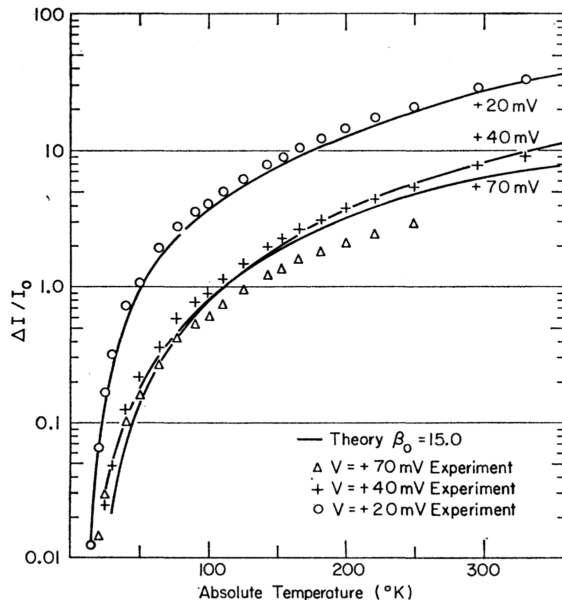

FIG. 4. The relative current change as a function of absolute temperature for several positive bias values. The solid lines were calculated with $\beta_0=15.0$.

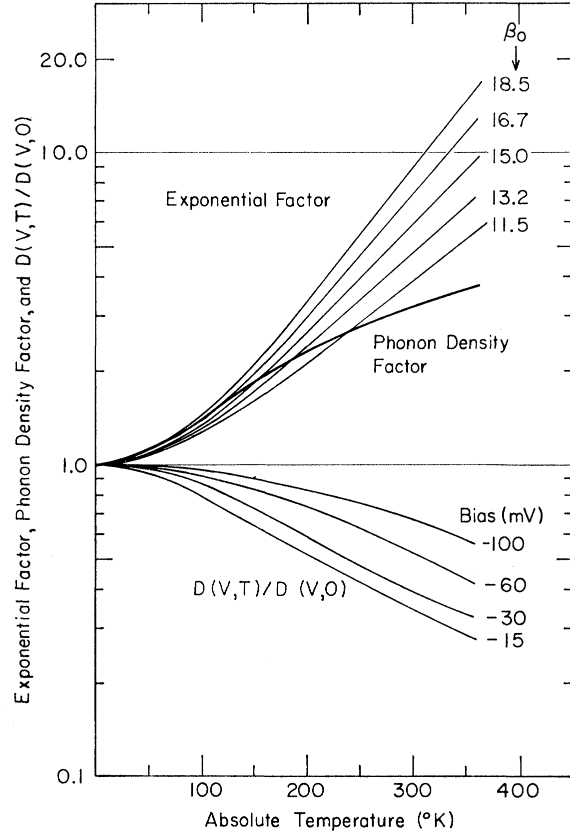

FIG. 5. Temperature dependence of various factors entering the approximate expression for the relative current change. [See Eq. (11) of text.]

change of the total current with temperature is then

$$\frac{\Delta I}{I_0}=\exp\left[\frac{2}{3}\left(\frac{E_g(0)}{\bar{E}_\perp(0)}-\frac{E_g(T)}{\bar{E}_\perp(T)}\right)\right]\frac{D(V,T)}{D(V,0)}\times\frac{\sum_i B_i(2N_i+1)}{\sum_i B_i}-1. \tag{11}$$

From the I-V characteristic at helium temperatures we obtained $B_2/B_1=3.07$, where the subscripts 1 and 2 refer to the TA and LA phonons, respectively.[21] The changes with temperature of the exponential factor, $D(V,T)/D(V,0)$, and the phonon density factor are shown in Fig. 5. The predictions of Eq. (11) at $V=-100$ mV, which is a favorable bias value for the approximate calculation, are shown together with the results of the machine calculations in Fig. 6 for $\beta_0=15.0$. The results of the approximate calculation lie only about 17% lower than those of the machine calculation. This discrepancy is approximately independent of temperature. This close

[21] From the I-V characteristic at helium temperatures one obtains actually $P_2(re)/P_1(re)=3.07$. In this approximate calculation we ignore the phonon energies in the tunneling probabilities of Eq. (4). Hence with Eq. (9) one obtains $P_2/P_1=B_2/B_1$.

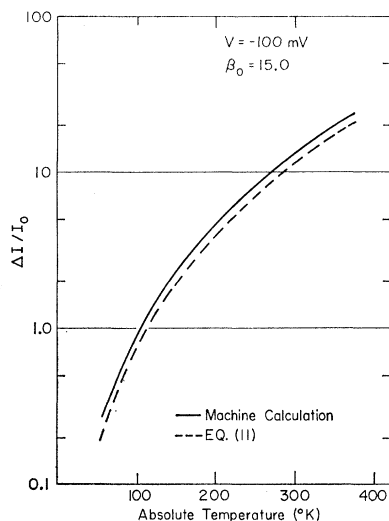

FIG. 6. Comparison of the machine calculation based on Eq. (1) with the approximate calculation based on Eq. (11) for $V=-100$ mV and $\beta_0=15.0$.

agreement justifies the following conclusions. (1) Because of the large value of β_0, the major contribution to $\Delta I/I_0$ comes from the change of E_g with temperature. (2) The temperature dependence of the Fermi functions causes the current to decrease with increasing temperature less strongly for larger reverse biases. This effect is shown in Fig. 5. (3) At small biases in both the forward and the reverse direction one sees experimentally a large increase in the ratio of $I(T)/I(0)$. This effect is due to the phonon structure, and it is not reproduced by Eq. (11). It is however, remarkably well produced by the machine calculation based on Eq. (1).

The agreement between the experimental data and the $\beta_0=15.0$ curves is particularly poor at the forward biases (see Fig. 4). The question arises whether one can expect a better agreement with a theory which does not describe accurately the shape of the I-V characteristic. In order to study this, we repeated the calculations using S integrals of the form

$$S_i(fe)=\int f_1(E_1)[1-f_2(E_2)](E_1E_2)^{1/2}dE, \quad (12)$$

and also of the form

$$S_i(fe)=\int f_1(E_1)[1-f_2(E_2)] \times[1-\exp(-2E_s/\bar{E}_\perp)]dE. \quad (13)$$

Equation (12) is derived from Esaki's density of states integral, and Eq. (13) is derived from Kane's D integrals for direct tunneling. In Eq. (12), E_s is the smaller of the two values E_1 and E_2.

The shapes of the I-V characteristic at $T=0$ obtained with these two expressions differ strongly for forward biases. Esaki's expression yields a pronounced peak; whereas Kane's expression yields an extented flat top region in the forward I-V characteristic. In spite of this difference in the shape of the I-V curves, the $\Delta I/I_0$ values are changed only slightly. Eq. (12) yields 10% lower values of $\Delta I/I_0$ at $V=+70$ mV and 3% higher values at $V=+20$ mV than Eq. (13). The two calculations agreed to within 3% for the reverse biases. We therefore conclude that the results of our calculations are insensitive to the detailed shape of the I-V characteristic.

The disagreement seen in Fig. 4 at $+70$ mV may be caused by other factors. At forward biases the correct knowledge of the Fermi level penetrations becomes increasingly important. We used the values calculated from the carrier concentration in the bulk material. The actual values in the junction region, however, are expected to be somewhat lower than in the bulk because of diffusion during the alloying process. Meyerhofer *et al.*[8] preferred values of the impurity concentrations near the junction about half of those in the bulk. This choice led also to a better agreement between the experimental and theoretical values of the tunneling exponent in Ref. 9. Smaller values of the Fermi functions will bring the theoretical curve to closer agreement with the data at $+70$ mV bias. Furthermore, variations in the Fermi level penetration from point to point due to statistical fluctuations in the impurity concentrations will also be more important in this bias region.

The deviations between theory and experiment at the other biases are not considered to be significant.

D. Absolute Magnitude of Tunneling Current

Although the determination of β_0 from the temperature dependence is based on several assumptions and approximations, the value $\beta_0=15.0\pm1$ for our diodes seems to be established well enough to justify an absolute magnitude comparison of the tunneling current with Kane's theory. The comparison is made for a current of 2.9×10^{-7} A which was observed at the bias voltage of $+20$ and -15 mV. At these biases only the TA phonon with $\hbar\omega=7.6\times10^{-3}$ eV is being emitted. The expression for the tunneling current at $T=0$ between the four (111) conduction band valleys and the valence band is

$$I=A\frac{\bar{E}_\perp{}^2(m_{x1}m_{y1}m_{z1}m_{x2}m_{y2}m_{z2})^{1/2}B}{2^{3/4}\pi^{5/2}\hbar^{13/2}m_{rx}{}^{1/4}E_g{}^{3/4}F^{1/2}}D \times\exp\left[-\frac{2}{3}\frac{E_g}{\bar{E}_\perp}\left(1\mp\frac{\hbar\omega}{E_g}\right)^{3/2}\right]. \quad (14)$$

Since $\zeta_p\gg\zeta_n, \bar{E}_\perp$, the D integral at small biases reduces to

$$D=(eV\mp\hbar\omega)-\bar{E}_\perp\exp\left(-\frac{\zeta_n}{\bar{E}_\perp}\right) \times\left[\exp\left(\frac{eV\mp\hbar\omega}{\bar{E}_\perp}\right)-1\right]. \quad (15)$$

From Eq. (8) and $\beta_0=15.0$ one obtains $\bar{E}_\perp=0.033$ eV. Using this value for $\bar{E}_\perp$ and $m_{rx}=0.03m_0$, the junction field F is determined by using Eq. (3). Using the effective mass values as determined by cyclotron resonance measurements one obtains $(m_{x1}m_{y1}m_{z1}m_{x2}m_{y2}m_{z2})^{1/2}=8\times10^{-4}m_0^3$. However, Spitzer *et al.*[22] observed in highly doped germanium effective masses which are larger than the cyclotron resonance masses. For the concentrations present in our diodes this amounts to an increase in the electron masses by approximately a factor of 1.2 and in the hole masses by a factor of 1.5. A more realistic estimate of the effective masses, therefore, yields $m_{rx}=0.045m_0$ and $(m_{x1}m_{y1}m_{z1}m_{z2}m_{y2}m_{z2})^{1/2}=2\times10^{-3}m_0^3$.

With $E_g=0.74$ eV at $T=0$ and the junction area $A=2\times10^{-3}$ cm², we obtain

$$I=1.5\times10^{61}DB\exp\left[-\beta_0\left(1\mp\frac{\hbar\omega}{E_g}\right)^{3/2}\right], \qquad (16)$$

where D is measured in units of erg and B, the coupling constant for the TA phonon, in units of erg² cm³ to yield I in A. Calculating B from Eq. (16), and the measured current $I=2.9\times10^{-7}$ A, one obtains

$$B=(8.3_{-5.1}^{+14.0})\times10^{-48}\text{ erg}^2\text{ cm}^3 \quad \text{at} \quad V=20\text{ mV},$$
$$B=(13.7_{-8.1}^{+24.1})\times10^{-48}\text{ erg}^2\text{ cm}^3 \quad \text{at} \quad V=-15\text{ mV}.$$

The uncertainty of these values results from the estimated error in the determination of β_0. It can be shown that random fluctuations in the impurity concentration cause an uncertainty in the absolute magnitude comparison which is smaller than that quoted. The values of B obtained at the two bias values differ slightly because the theory fails to predict correctly the shape of the I-V characteristic.

Kane[5] estimated a value $B=4.3\times10^{-49}$ erg² cm³ for scattering from the (000) to the (111) conduction band minimum from optical absorption measurements.[14] This value is between 10 and 40 times smaller than that determined from Eq. (14). This is too large a discrepancy to be accounted for by the fact that in the case of interband tunneling the scattering takes place between band extrema at **k** values near the middle of the forbidden gap. It is more likely that Kane's expression for the absolute magnitude is not applicable in its present form to the case of Sb-doped Ge.

IV. SUMMARY AND CONCLUSIONS

The temperature dependence of the tunneling current in Sb-doped germanium tunnel diodes was found to be in good agreement with the present theory of indirect tunneling. The manner in which $\Delta I/I_0$ was compared with the theory is quite insensitive to the detailed shape of the I-V characteristic, and, hence, to the exact form of the integrals over the electron distribution functions.

The agreement between theory and experiment is, however, very sensitive to the correct choice of the tunneling exponent β_0 [see Eq. (8)] and of the relative strengths of the different contributions to the net current across the junction. The value $\beta_0=15.0\pm1$ determined from the temperature dependence is in agreement with the value determined from the effect of stress on the same diodes within the quoted uncertainties.

The strengths of the phonon contributions measured by $P_1(fe)$, $P_1(re)$, $P_2(fe)$, and $P_2(re)$ were found to have the ratios 1:1.0:3.07:3.07, respectively. These ratios agree with the ratios of the conductance changes in the I-V characteristic near the phonon thresholds at 1.2°K They also agree with the findings of Hall[23] if the contributions of the optical phonons are taken together with the LA phonon.

It is somewhat surprising that $P(fe)=P(re)$ for both phonons in view of the second term in the exponent of Eq. (4) which changes sign as one goes from forward to reverse current flow. With our value $\beta_0=15.0$ for the tunneling exponent, this term amounts to a factor of 1.6 for the TA phonon between the reverse and forward directions, and a factor of 5.9 for the LA phonon-assisted current. If we accept these factors as being correctly included, then the observed equality $P(fe)=P(re)$ implies that there must be a compensating asymmetry in the electron-phonon coupling constants $B_1(fe)\neq B_1(re)$ and $B_2(fe)\neq B_2(re)$. Although some difference in the electron-phonon coupling constants is expected because of the difference in electron shielding on the N- and P-type side of the junction, it would be surprising if these differences were just of the correct sign and magnitude to cancel the asymmetries for both types of phonons independently. It seems, therefore, probable that the phonons are not adequately incorporated into the theory. We know of no experiment which has been able to check the predicted asymmetry of the tunneling exponent with respect to forward and reverse current flow.

If the variation for pure germanium is assumed for the band gap, the magnitude of the temperature dependence can be related to the value of the tunneling exponent, and there is sufficient sensitivity at higher temperatures to determine the exponent to within $\pm7\%$. This uncertainty is equivalent to the entire effect expected from the explicit temperature dependence of the effective masses. We can, therefore, only say that our results do not conflict with the temperature dependence of the effective masses that is expected theoretically. Since the temperature dependence of the effective masses gives rise to only a minor variation in the tunneling exponent, our conclusions will not be changed very much even if there is a significant departure from the theoretically expected behavior.

Using the experimentally observed value of the

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tunneling exponent, we found the observed tunneling current to be about 10–40 times larger than that predicted by Kane's expression. This discrepancy seems too large to be accounted for by the uncertainty in the magnitude of the electron-phonon coupling constant.

It should be emphasized that the good quantitative agreement between the temperature dependence of the tunneling current calculated from Kane's theory of indirect tunneling and the experimental data does not prove the correctness of Kane's expression. It merely indicates that the coefficient of $Eg^{3/2}m^{*1/2}/F$ which appears in the exponent is at least approximately correct. Since our comparison is insensitive to the shape of the I-V characteristic, we cannot offer any evidence relating to the structure of the D function. Furthermore, no evidence for the asymmetry of the tunneling exponent with respect to forward and reverse current flow was observed.

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Microwave Conductivity of Semiconductors in the Presence of High Steady Electric Fields

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The distribution function of carriers in a semiconductor when subjected to a small microwave field and a high steady electric field is derived, considering both the acoustic and optical phonon scattering. Expressions for microwave conductivity and change in apparent dielectric constant are obtained from the distribution function. It is shown by numerical calculations that the conductivity evaluated from these expressions agree closely with the experimental value. The calculated value of the change in apparent dielectric constant, however, is found to be of the same order as the experimental value, but the agreement is poorer than that for the conductivity.

I. INTRODUCTION

THE microwave conductivity of semiconductors in the presence of high steady electric fields has been studied by several workers. In the first experiments reported by Arthur *et al.*[1] the attenuation of a microwave signal produced by the sample of known dimensions in the presence of a steady field was measured. The attenuation so produced was assumed to be proportional to the slope mobility and the experimental data were used to derive the high field conductivity. This assumption is apparently justified if the product of the microwave frequency and the momentum relaxation time is negligible compared to unity, as was the case in these experiments.

Later experiments carried out by Gibson *et al.*,[2] however, show that the microwave mobility in the presence of high steady fields is not the same as the slope mobility but is intermediate between the slope mobility and the dc mobility. This result that the microwave mobility is different from the slope mobility even when the product of microwave frequency and momentum relaxation time is much less than unity may be explained when the mobility expressions applicable to the problem are properly developed. Such expressions have been derived by Paranjape[3] and also by Gibson *et al.*[2]

It has been assumed that the carrier density is high enough to produce a Maxwellian energy distribution due to predominant interelectronic collisions. But, the carrier temperature which is determined by the energy and momentum balance conditions, is higher than the temperature of the lattice. It is then shown that the perturbation in the carrier temperature produced by the microwave signal is not in phase with the signal, but leads it. The lead angle is determined by the applied steady field and the product of the microwave frequency and the energy relaxation time, rather than the momentum relaxation time. Since this product is comparable to unity at the experimental frequencies, the perturbed temperature of the carriers differs appreciably in phase from the microwave signal. Hence, the microwave conductivity is much different from that derived from the slope of the conductivity versus field curves; also, a

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